# Failure of parameter identification based on adaptive synchronization techniques

Wei Lin* and Huan-Fei Ma

*Key Laboratory of Mathematics for Nonlinear Sciences (Fudan University), Ministry of Education, Research Center for Nonlinear Sciences, School of Mathematical Sciences, Fudan University, Shanghai 200433, China*

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In this paper, several examples as well as their numerical simulations are provided to show some possible failures of parameter identification based on the so-called adaptive synchronization techniques. These failures might arise not only when the synchronized orbit produced by the driving system is designed to be either some kind of equilibrium or to be some kind of periodic orbit, but also when this orbit is deliberately designed to be chaotic. The reason for emergence of these failures is theoretically analyzed in the paper and the boundedness of all trajectories generated by the coupled systems is rigorously proved. Moreover, synchronization techniques are proposed to realize complete synchronization and unknown parameter identification in a class of systems where nonlinear terms are not globally Lipschitz. In addition, unknown parameter identification is studied in coupled systems with time delays.

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## I. INTRODUCTION

A classical phenomenon related to synchronization is owing to Huygens' observation about the synchrony of pendulum clocks [1]. Since this historical discovery, synchronization as an omnipresent technical issue has become a topic of importance in numerous applications. Moreover, the basic concept referring to chaos synchronization in coupled chaotic systems was initially introduced by Pecora and Carrol in 1990 [2]. Since their seminal paper, chaos synchronization as an interesting research topic of great potential application has been widely investigated and consequently applied in plenty of fields, ranging from secure communications to pattern recognitions, from complex network dynamics to optimization of nonlinear systems, and even from chemical reaction to brain activity analysis [3]. In particular, various synchronization approaches, such as conventional linear or nonlinear feedback coupling techniques, impulsive coupling method, invariant manifold method, adaptive design coupling techniques, and white-noise-based coupling have been fruitfully proposed [4,5], and several types of synchronization, such as complete synchronization, generalized synchronization, phase synchronization, and lag synchronization, have been introduced [6–9].

Among all the proposed approaches for realization of complete synchronization in coupled chaotic systems with or without time delays, the newly developed technique based on the adaptive coupling has aroused a great amount of attention from many researchers [10–18]. Their theoretical and numerical explorations have shown that unknown parameters could be accurately identified in some neural network models with or without time delays and in several well-known chaotic systems where nonlinear terms are not globally Lipschitz. More precisely, they consider an $n$-dimensional nonlinear system in the form of

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{p}), \tag{1}$$

where

$$\mathbf{x} = (x_1,x_2,\ldots,x_n)^T \in \mathbb{R}^n,\ \mathbf{F}(x,\mathbf{p}) = (F_1(\mathbf{x},\mathbf{p}),F_2(\mathbf{x},\mathbf{p}),\ldots,F_n(\mathbf{x},\mathbf{p}))^T,$$

and

$$F_i(\mathbf{x},\mathbf{p}) = c_i(\mathbf{x}) + \sum_{j=1}^{m} p_{ij}f_{ij}(\mathbf{x}), \quad i = 1,2,\ldots,n. \tag{2}$$

Moreover, each $c_i(\mathbf{x})$, $f_{ij}(\mathbf{x})$ is assumed to be a real valued function, $\mathbf{p}=\{p_{ij}\} \in \mathcal{U} \subset \mathbb{R}^n$ are $(nm)$ parameters to be identified, and $\mathcal{U}$ is some bounded set. Thus, an interesting question arises: "Is it possible to accurately identify all the $(nm)$ parameters $\mathbf{p}$ in system (1) if only the bounded driving signal $\mathbf{x}(t)$ generated by this system is experimentally obtained?" As said above, the answer to this question is reportedly positive when the response system with adaptive coupling is designed as

$$\dot{\mathbf{y}} = \mathbf{F}(\mathbf{y},\mathbf{q}) + \boldsymbol{\epsilon}\cdot\mathbf{e},$$
$$\dot{\epsilon}_i = -r_i e_i^2, \quad \dot{q}_{ij} = -\delta_{ij}e_i f_{ij}(\mathbf{y}),$$
$$i = 1,2,\ldots,n, \quad j = 1,2,\ldots,m, \tag{3}$$

where the feedback coupling $\boldsymbol{\epsilon}\cdot\mathbf{e}=(\epsilon_1 e_1,\epsilon_2 e_2,\ldots,\epsilon_n e_n)^T$, each error $e_i=(y_i-x_i)$, $\mathbf{q}=\{q_{ij}\}$, and each $r_i$, $\delta_{ij}$ is an arbitrarily positive constant. The existing numerical results have shown that the complete synchronization between system (1) and (3) could always be achieved, and that the varying parameters $\mathbf{q}$ in Eqs. (3), initiating from arbitrary values, will be asymptotically convergent to the accurate values of the parameters $\mathbf{p}$ as time tends towards positive infinity. Seemingly, their theoretical arguments are based on a delicate design for the Lyapunov function, on the well-known Lyapunov stability theorem, and even on the LaSalle invariance principle. As a matter of fact, the parameter identification might always be failed in real application if those func-

*Author to whom correspondence should be addressed. Mailing address: School of Mathematical Sciences, Fudan University, 220 HanDan Rd., Shanghai 200433, China. FAX: 86-21-6564-2342. Email address: wlin@fudan.edu.cn

 

tions with unknown parameters to be identified are designed to be mutually linearly dependent or approximately linearly dependent on the synchronized orbit in the synchronization manifold. Here, and throughout the paper, the synchronized orbit in the synchronization manifold is actually some positive limit set of the driving signal generated by system (1).

The aim of this paper is not only to provide several concrete examples showing some possible failures of parameter identification, but also to explain the reason for emergence of these failures. Besides, the following argument in the paper will show that *chaotic property of the synchronized orbit in the synchronization manifold is not always crucial to an achievement of parameter identification*. However, a proper utilization of chaotic property might lead to success in parameter identification.

The rest of the paper is organized as follows. In Sec. II, three concrete examples with their numerical simulations are, in succession, given to illustrate the possible failures of parameter identification. The synchronized orbits in these examples are designed to be some equilibrium, periodic oscillation, and chaotic attractor, respectively. In Sec. III, the reason for occurrence of these failures, as well as the bounded property of all trajectories generated by the coupled systems (1) and (3), is theoretically expatiated. Furthermore, in Sec. IV, synchronization techniques are further proposed to realize both complete synchronization and unknown parameter identification in a class of polynomial systems where nonlinear terms are not globally Lipschitz. In Sec. V, parameter identification is investigated in coupled systems with time delays. Finally, the paper is closed with some concluding remarks.

## II. EXAMPLES SHOWING FAILURE OF PARAMETER IDENTIFICATION

In this section, three groups of driving-and-response systems are concretely presented to somewhat show occurrence of failed parameter identification.

First, let us consider the Lorenz system:

$$\dot{x}_1 = p_1(x_2 - x_1),$$

$$\dot{x}_2 = p_2 x_1 - x_1 x_3 - x_2,$$

$$\dot{x}_3 = x_1 x_2 - p_3 x_3 \tag{4}$$

as a driving system, where each $p_i$ is a parameter to be identified. Then, the corresponding response system becomes

$$\dot{y}_1 = q_1(y_2 - y_1) + \epsilon_1(y_1 - x_1),$$

$$\dot{y}_2 = q_2 y_1 - y_1 y_3 - y_2 + \epsilon_2(y_2 - x_2),$$

$$\dot{y}_3 = y_1 y_2 - q_3 y_3 + \epsilon_3(y_3 - x_3), \tag{5}$$

where the updating laws of $\mathbf{q}=(q_1,q_2,q_3)$ and $\boldsymbol{\epsilon}=(\epsilon_1,\epsilon_2,\epsilon_3)$ are selected as the following: $\dot{q}_1=-\delta_1(y_1-x_1)(y_2-y_1)$, $\dot{q}_2=-\delta_2(y_2-x_2)y_2$, $\dot{q}_3=-\delta_3(y_3-x_3)(-y_3)$, $\dot{\epsilon}_1=-r_1(y_1-x_1)^2$, $\dot{\epsilon}_2=-r_2(y_2-x_2)^2$, and $\dot{\epsilon}_3=-r_3(y_3-x_3)^2$.

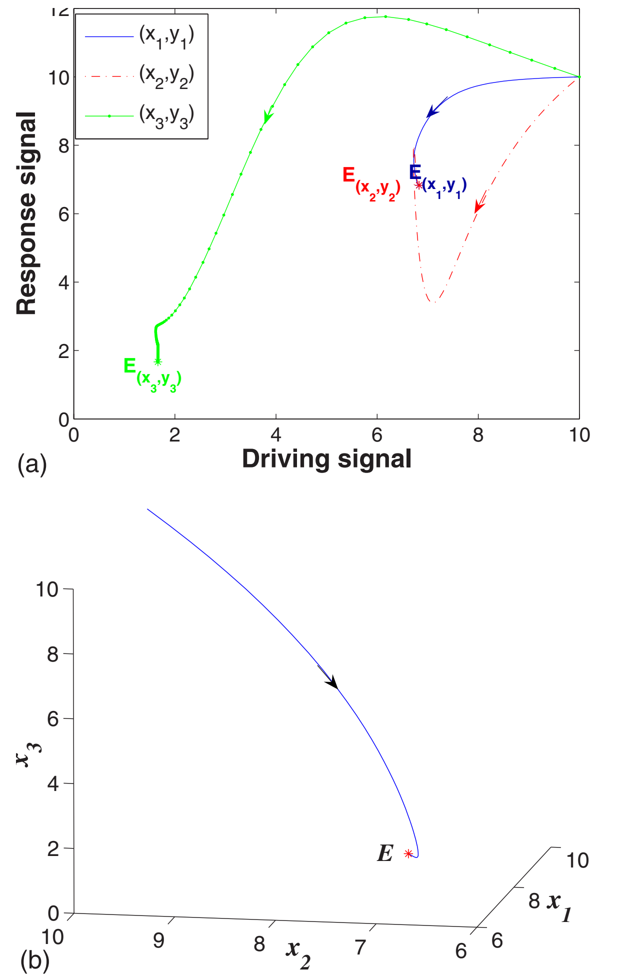


FIG. 1. (Color online) A successful complete synchronization between the Lorenz systems (4) and (5) by means of the adaptive design coupling. Here, system (4) possesses an asymptotically stable equilibrium $E=(6.8313,6.8313,1.6667)^T$ instead of the strange attractor. The variation of the driving signal with the response state are shown in (a) and the evolution of response state in the phase plane are depicted in (b). Here, $r_i=15$, $\delta_i=2$, and all the initial values are simply chosen as $x_i^0=y_i^0=10$, $q_i^0=1$, $\epsilon_i^0=1$ ($i=1,2,3$).

In particular, when the parameters are chosen as $p_1=35$, $p_2=\frac{8}{3}$, and $p_3=28$, the complete synchronization between systems (4) and (5) could be easily achieved, which is numerically shown in Fig. 1(a). These specified parameters, which are different from the classical parameters inducing chaotic attractor of the Lorenz system, simply make the synchronized orbit become an asymptotically stable equilibrium $E$ of system (4), as is shown in Fig. 1(b). If the reported analytical results are completely correct, it could be expected that the varying parameters $\mathbf{q}(t)=(q_1(t),q_2(t),q_3(t))$ will be eventually convergent to the accurate values of the parameters $\mathbf{p}=(p_1,p_2,p_3)$. Nevertheless, contrary to the expectation, the varying parameters $q_1(t)$, initiating from almost every value, does not approach the accurate value of $p_1$. As depicted in Fig. 2, although the values of $q_{2,3}(t)$ tend to the values of $p_{2,3}$ with time evolution, $q_1(t)$ always keeps a distance from the accurate value of $p_1$. This numerical result

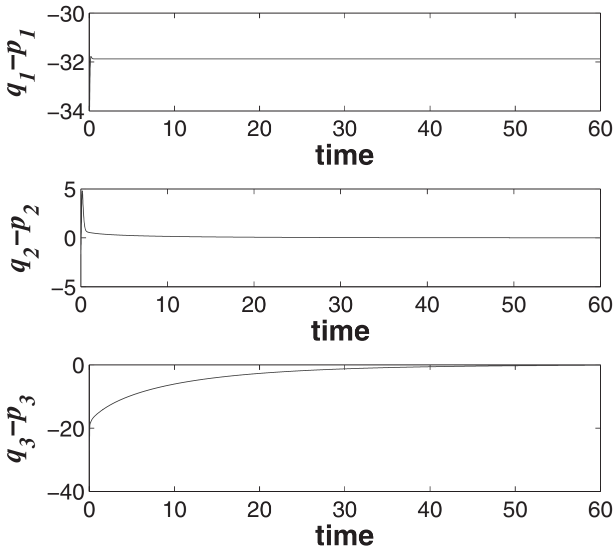


FIG. 2. The variation of the error between the parameters $q_i$ and $p_i$ with time initiating from 0 to 60 with stepsize 0.01 ($i=1,2,3$). In particular, $q_1$ fails to identify the accurate value of $p_1$. All the parameters and initial values for coupling systems are the same as those given in Fig. 1.

clearly implies the occurrence of failed parameter identification for $q_1$. Intuitively, there must exist an intrinsic reason for such a difference between $q_1$ and $q_{2,3}$ when adaptive synchronization techniques are taken into account.

Secondly, introduce a driving system, based on the Chen system, in the form of

$$\dot{x}_1 = 30(x_2 - x_1) + \mathcal{Q}(x_1,x_2,p_1,p_2),$$

$$\dot{x}_2 = (28-30)x_1 - x_1x_3 + 28x_2,$$

$$\dot{x}_3 = x_1x_2 - 3x_3, \tag{6}$$

where the additional term

$$\mathcal{Q}(x_1,x_2,p_1,p_2) = 0.1 \times \left\{ p_1 \frac{[x_1\cos(0.9026) + x_2\sin(0.9026)]^2}{23.44^2} - p_2\left[\frac{[-x_1\sin(0.9026)+x_2\cos(0.9026)]^2}{7.19^2} - 1\right]\right\},$$

and both $p_1$ and $p_2$ are parameters to be identified. As a matter of fact, without the term $\mathcal{Q}$, system (6) becomes the original Chen system, producing an attractive periodic orbit. As displayed in Fig. 3, the projection of this periodic orbit into the $x_1$-$x_2$ plane is approximately looked upon as an ellipse. Thus, when $p_1=1$ and $p_2=-1$, the term $\mathcal{Q}$ actually is an approximate formula of this projection in the $x_1$-$x_2$ plane.

Given the driving signal $(x_1,x_2,x_3)^T$ generated by system (6), we introduce a response system in the form of

$$\dot{y}_1 = 30(y_2-y_1) + \mathcal{Q}(y_1,y_2,q_1,q_2) + \epsilon_1(y_1-x_1),$$

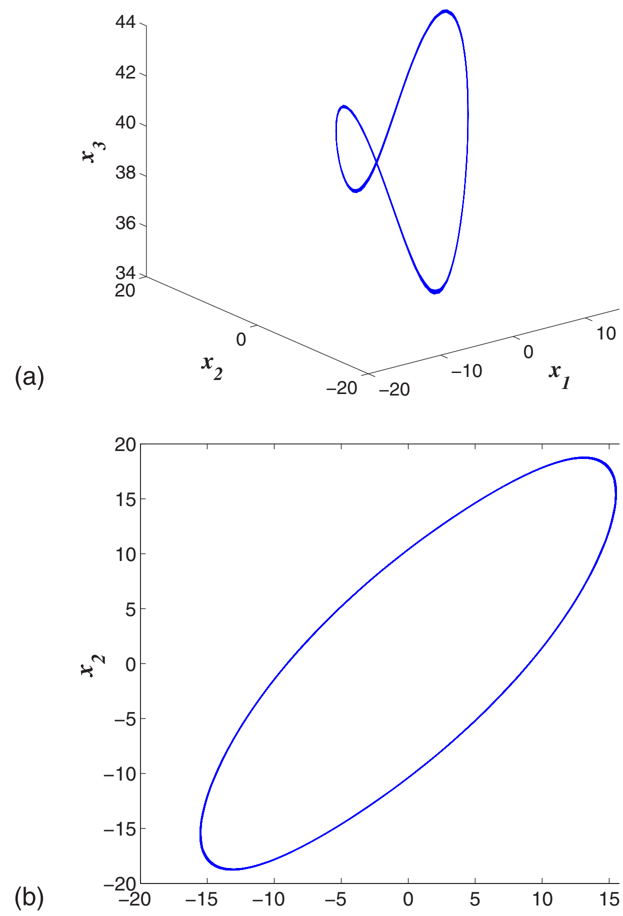


FIG. 3. (Color online) The attractive periodic orbit generated by the original Chen's system [system (6) when $\mathcal{Q}\equiv 0$]. The periodic orbit in the $x_1$-$x_2$-$x_3$ phase plane (a) and its projection in the $x_1$-$x_2$ plane (b).

$$\dot{y}_2 = (28-30)y_1 - y_1y_3 + 28y_2 + \epsilon_2(y_2-x_2),$$

$$\dot{y}_3 = y_1y_2 - 3y_3 + \epsilon_3(y_3-x_3). \tag{7}$$

Here, according to (3), the updating laws of the varying parameters are taken as

$$\dot{q}_1 = -\delta_1(y_1-x_1) \times \left[\frac{[y_1\cos(0.9026)+y_2\sin(0.9026)]^2}{23.44^2}\right],$$

$$\dot{q}_2 = -\delta_2(y_1-x_1)\left[-\frac{[-y_1\sin(0.9026)+y_2\cos(0.9026)]^2}{7.19^2} + 1\right];$$

the adaptive techniques of the coupling strengths are chosen as $\dot{\epsilon}_1=-r_1(y_1-x_1)^2$, $\dot{\epsilon}_2=-r_2(y_2-x_2)^2$, and $\dot{\epsilon}_3=-r_3(y_3-x_3)^2$.

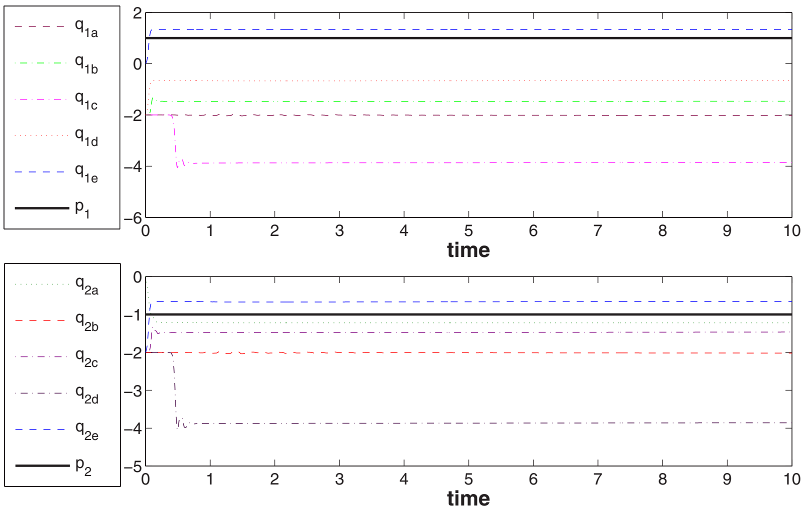


FIG. 4. (Color online) The variation of $q_i$ ($i=1,2$) with time initiating from 0 to 10 with stepsize 0.01 when its initial value $q_{i\varrho}$ is taken differently ($\varrho=a,b,c,d,e$). Here, $r_j=2$, $\delta_j=1$, and all the initial values are taken as $x_j^0=y_j^0=10$, $\epsilon_j^0=1$, $j=1,2,3$.

Analogously, it is contrary to the expectation that both $q_1(t)$ and $q_2(t)$, starting from randomly selected values, fail to approach the parameters $p_1=1$ and $p_2=-1$, as shown in Fig. 4. This example, as well as the first example, reinforces a fact that failure of parameter identification based on adaptive synchronization techniques does occur when the synchronized orbit is particularly designed to be some kind of steady dynamics, such as asymptotically stable equilibrium and attractive periodic orbit.

Instead of the above-mentioned steady orbit in the synchronization manifold, the existing numerical results [10–18] always show that parameter identification could be surely achieved when the synchronized orbits are designed to be chaotic in advance. Then, it becomes interesting to ask such a question: "Is chaotic property of the synchronized orbit in the synchronization manifold necessary to the achievement of parameter identification based on the adaptive techniques?"

To find an answer to this question, consider a four-dimensional driving system,

$$\dot{x}_1 = p_1(x_2 - x_1),$$

$$\dot{x}_2 = p_2 x_1 - x_1 x_3 - x_2,$$

$$\dot{x}_3 = x_1 x_2 - p_3 x_3 + p_4 x_3(1 + x_4^3),$$

$$\dot{x}_4 = a x_4 + b(x_1 - x_3), \tag{8}$$

which is developed from the original chaotic Lorenz system. Here, $p_1=10$, $p_2=28$, and $p_3=\frac{8}{3}$ are the three special parameters for the original Lorenz system to generate chaotic attractor; $a=-100$, $b=0.1$, and $p_4=1$. Given these specified parameters, the orbit produced by system (8) in the synchronization manifold still exhibits chaotic character in the phase plane, which is displayed by Figs. 5(a) and 5(b). This chaotic character is further verified by calculating the largest Lyapunov exponent of the system (namely, $\lambda_1 \approx 0.54274 > 0$), as is shown in Fig. 5(c).

Provided with the driving signal produced by system (8), the complete synchronization between systems (8) and its response system could be numerically achieved as long as the response system is designed as

$$\dot{y}_1 = q_1(y_2 - y_1) + \epsilon_1(y_1 - x_1),$$

$$\dot{y}_2 = q_2 y_1 - y_1 y_3 - y_2 + \epsilon_2(y_2 - x_2),$$

$$\dot{y}_3 = y_1 y_2 - q_3 y_3 + q_4 y_3(1 + y_4^3) + \epsilon_3(y_3 - x_3),$$

$$\dot{y}_4 = a y_4 + b(y_1 - y_3) + \epsilon_4(y_4 - x_4), \tag{9}$$

in which the updating laws of the parameters are taken as $\dot{q}_1=-\delta_1(y_1-x_1)(y_2-y_1)$, $\dot{q}_2=-\delta_2(y_2-x_2)y_1$, $\dot{q}_3=-\delta_3(y_3-x_3)\times(-y_3)$, and $\dot{q}_4=-\delta_4(y_3-x_3)[y_3(1+y_4^3)]$; the adaptive coupling strengths are taken as $\dot{\epsilon}_1=-r_1(y_1-x_1)^2$, $\dot{\epsilon}_2=-r_2(y_2-x_2)^2$, $\dot{\epsilon}_3=-r_3(y_3-x_3)^2$, and $\dot{\epsilon}_4=-r_4(y_4-x_4)^2$. In spite of the success in complete synchronization and in parameter identification for $q_{1,2}(t)$, it is impossible to utilize $q_{3,4}(t)$, initiating from almost every value, to accurately identify the parameters $p_{3,4}$ in system (8). All these are shown in Fig. 6. Clearly, this numerical example implies a negative answer to the above-posed question.

*Remark*. The fourth-order Runge-Kutta scheme is used to solve all the ordinary differential equations in our numerical simulations.

## III. REASON FOR FAILED PARAMETER IDENTIFICATION

Three concrete examples in the preceding section have shown that unknown parameter identification might be failed no matter what kind of dynamical phenomenon is displayed in the synchronization manifold. However, many existing numerical results always show successful unknown parameter identification. In order to clarify this paradox, we, in what follows, perform a more delicate argument by adopting the LaSalle invariance principle [19] and the particular properties of an autonomous system. Similar to [11], we set a Lyapunov function candidate as

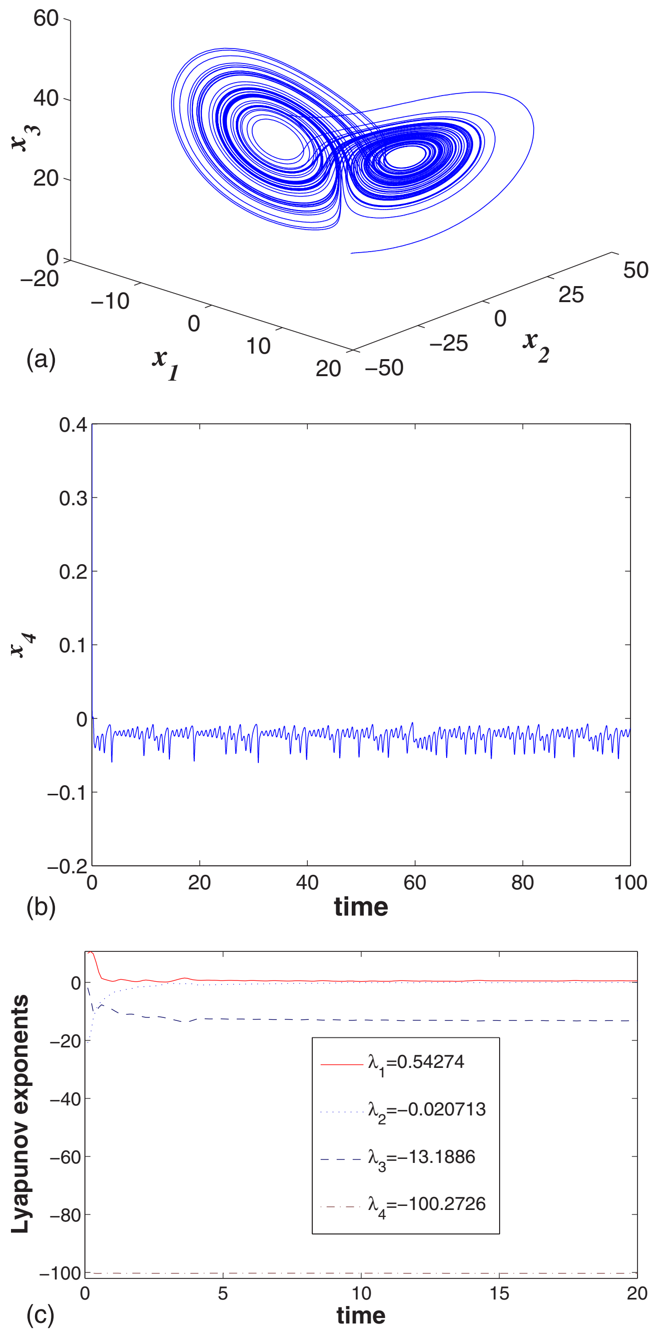


FIG. 5. (Color online) The strange attractor produced by system (8) are, respectively, plotted in the $x_1$-$x_2$-$x_3$ plane (a) and in the time-state-$x_4$ plane (b). The chaotic property is verified by the Lyapunov exponent portrait (c), where the largest Lyapunov exponent $\lambda_1$ is above zero.

$$V(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q}) = \frac{1}{2}\sum_{i=1}^{n} e_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\delta_{ij}}(q_{ij}-p_{ij})^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{r_i}(\epsilon_i+L)^2. \tag{10}$$

Then, the derivative of the function $V(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q})$ along with the coupled systems (1) and (3) could be estimated by

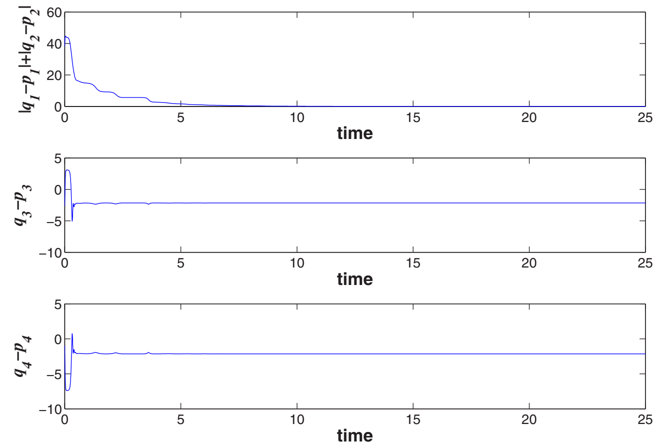


FIG. 6. (Color online) The variation of the error between the parameters $q_j$ and $p_j$ with time initiating from 0 to 25 with stepsize 0.01. Indeed, $q_{3,4}$ fails to identify the accurate value of $p_{3,4}$, respectively. Here, $r_j=15$, $\delta_j=2$, and all the initial values are taken as $x_j^0=1$, $y_1^0=6$, $y_2^0=30$, $y_3^0=10$, $y_4^0=1$, $q_j^0=0$, and $\epsilon_j^0=1$ $(j=1,2,3,4)$.

$$\dot{V}(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q}) \leqslant (nl-L)\sum_{i=1}^{n} e_i^2.$$

Here, it should be pointed out that $l$ is not a local Lipschitz constant of the function $F_i(\mathbf{x},\mathbf{p})$, but a uniform Lipschitz constant. This uniform Lipschitz condition is essential in the following argument, because the bounded property of the trajectory $\mathbf{y}(t)$ generated by the response system (3) are not confirmed yet, but waiting for confirmation.

Now, *we can contend that* $\mathbf{e}(t)$, $\boldsymbol{\epsilon}(t)$, *and* $\mathbf{q}(t)$ *are bounded for all* $t \geqslant t_0$, *where* $t_0$ *is the initial time*. The verification of this assertion is performed by contradiction. On the one hand, one of the three variables is supposed to be unbounded in the interval $[t_0,+\infty)$, so that $V(\mathbf{e}(t),\boldsymbol{\epsilon}(t),\mathbf{q}(t))$ is also unbounded in $[t_0,+\infty)$ according to Eq. (10). On the other hand, due to $\dot{V}(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q}) \leqslant 0$ for sufficiently large $L$, one has $V(\mathbf{e}(t),\boldsymbol{\epsilon}(t),\mathbf{q}(t)) \leqslant V(\mathbf{e}(t_0),\boldsymbol{\epsilon}(t_0),\mathbf{q}(t_0))$. This contradiction thus verifies the validity of our assertion. Also, it is concluded that $\mathbf{y}(t)=\mathbf{e}(t)+\mathbf{x}(t)$ is bounded for all $t \geqslant t_0$, since the driving signal $\mathbf{x}(t)$ is assumed to be bounded in advance.

Therefore, by virtue of the LaSalle invariance principle, any trajectory $(\mathbf{x}(t),\mathbf{y}(t),\boldsymbol{\epsilon}(t),\mathbf{q}(t))$ generated by the autonomous coupled systems (1) and (3) will eventually approach the largest invariant set, denoted by $\mathcal{M}$, contained in the set

$$\mathcal{E} = \{(\mathbf{x},\mathbf{y},\boldsymbol{\epsilon},\mathbf{q}) \mid \dot{V}(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q}) = 0\}.$$

Then, we aim to make a clear description of this invariant set $\mathcal{M}$ with respect to systems (1) and (3). Notice that $\dot{V}(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q})=0$ implies $\mathbf{e}=\mathbf{x}-\mathbf{y}=0$. Then, it could be easily concluded that $\mathbf{e}(t)=\mathbf{y}(t)-\mathbf{x}(t)\to 0$ as $t\to+\infty$. Also, for every orbit $(\mathbf{x}(t),\mathbf{y}(t),\boldsymbol{\epsilon}(t),\mathbf{q}(t)) \in \mathcal{M}$, $\mathbf{e}(t)\equiv 0$, $\dot{\epsilon}_i(t)\equiv 0$, and $\dot{q}_{ij}(t)\equiv 0$, which follows from the invariance of the orbit in $\mathcal{M}$.

In what follows, it is shown under what kind of condition each entry of $\boldsymbol{\epsilon}(t)$ and $\mathbf{q}(t)$ is surely convergent to some constant. The convergence of $\boldsymbol{\epsilon}(t)$ is quite obvious simply due to the monotonic and bounded property of each entry in $\boldsymbol{\epsilon}(t)$. As for the bounded $\mathbf{q}(t)$, there must exist a series $\{t_n\}_{n=0}^{\infty} \to +\infty$ such that $\mathbf{q}(t_n) \to \mathbf{q}^* = \{q_{ij}^*\}$, $\boldsymbol{\epsilon}(t_n) \to \boldsymbol{\epsilon}^*$, and $\mathbf{e}(t_n) \to \mathbf{0}$ as $n \to \infty$. By using the particular properties of the solution generated by autonomous differential equations, we have $\lim_{n\to\infty}\mathbf{q}(t+t_n) = \lim_{n\to\infty}\mathbf{q}(t;t_0,\mathbf{q}(t_n)) = \mathbf{q}(t;t_0,\mathbf{q}^*)$, $\lim_{n\to\infty}\mathbf{e}(t+t_n) = \mathbf{e}(t;t_0,\mathbf{0})$, and $\lim_{n\to\infty}\boldsymbol{\epsilon}(t+t_n) = \boldsymbol{\epsilon}(t;t_0,\boldsymbol{\epsilon}^*)$ for all $t$. Then, from the uniqueness of the trajectory and the invariance of $\mathcal{M}$, it follows that $\mathbf{q}(t;t_0,\mathbf{q}^*) \equiv \mathbf{q}^*$, $\mathbf{e}(t;t_0,\mathbf{0}) \equiv \mathbf{0}$, and $\boldsymbol{\epsilon}(t;t_0,\boldsymbol{\epsilon}^*) \equiv \boldsymbol{\epsilon}^*$.

Now, a combination of systems (1) and (3) yields

$$\begin{aligned}\dot{e}_i &= \dot{x}_i - \dot{y}_i = c_i(\mathbf{x}) - c_i(\mathbf{y}) + \sum_{j=1}^{m} p_{ij} f_{ij}(\mathbf{x}) \\ &\quad - \sum_{j=1}^{m} q_{ij} f_{ij}(\mathbf{y}) - \epsilon_i(y_i - x_i) = c_i(\mathbf{x}) - c_i(\mathbf{y}) \\ &\quad + \sum_{j=1}^{m} [p_{ij} f_{ij}(\mathbf{x}) - q_{ij} f_{ij}(\mathbf{x})] + \sum_{j=1}^{m} [q_{ij} f_{ij}(\mathbf{x}) - q_{ij} f_{ij}(\mathbf{y})] \\ &\quad - \epsilon_i(y_i - x_i). \end{aligned} \tag{11}$$

For any trajectory $\mathbf{e}(t;t_0,\mathbf{0}) \equiv \mathbf{0}$, $\boldsymbol{\epsilon}(t;t_0,\boldsymbol{\epsilon}^*) \equiv \boldsymbol{\epsilon}^*$, and $\mathbf{q}(t;t_0,\mathbf{q}^*) \equiv \mathbf{q}^*$, we have

$$\sum_{j=1}^{m} [p_{ij} - q_{ij}(t)] f_{ij}(\mathbf{x}(t)) = 0, \tag{12}$$

where each $q_{ij}(t)$ is identical to some constant $q_{ij}^*$, and $\mathbf{x} = \mathbf{x}(t)$ is the synchronized orbit in the synchronization manifold. Thus, a question appears: "Is each $q_{ij}^*$ surely equal to $p_{ij}$?" From Eq. (12), the answer to this question is theoretically positive provided: *for any given* $i$, $\{f_{ij}(\mathbf{x}), j = 1,2,\ldots,m\}$ *are linearly independent on the synchronized orbit* $\mathbf{x} = \mathbf{x}(t)$ *in the synchronization manifold* [LIM].

For an accurate definition of linearly independent or linearly dependent functions, refer to [20]. Also, it is valuable to mention that two linearly independent functions in a domain could be linearly dependent in some subset contained in this domain. For example, functions $g_1(s,u) = s$ and $g_2(s,u) = u^2$ are obviously linearly independent in $\mathbb{R}^2$ but they are linearly dependent in a parabolalike subset $\mathcal{S}_\mu = \{(s,u) \in \mathbb{R}^2 | s = \mu u^2\} \subset \mathbb{R}^2$ for some nonzero constant $\mu$.

Therefore, it is concluded that if hypothesis [LIM] is satisfied, every trajectory generated by the coupled systems (1) and (3) will eventually approach the invariant set

$$\mathcal{M} = \{(\mathbf{x},\mathbf{y},\boldsymbol{\epsilon},\mathbf{q}) | \mathbf{e} = \mathbf{x} - \mathbf{y} = \mathbf{0},\ \epsilon_i = \epsilon_i^*,\ q_{ij} = p_{ij}\},$$

where each $\epsilon_i^*$ is a constant depending on the initial values of the trajectory. This thus implies a successful parameter identification.

If hypothesis [LIM] is not satisfied, there at least exist some $i = i_0$ and $j = j_1, j_2$ such that either (a) two nonzero functions $f_{i_0 j_1}(\mathbf{x})$ and $f_{i_0 j_2}(\mathbf{x})$ are linearly dependent on the synchronized orbit $\mathbf{x}(t)$, or that (b) $f_{i_0 j_1}(\mathbf{x}(t)) \equiv 0$. Accordingly, in case (a), $f_{i_0 j_1}(\mathbf{x}(t)) = c f_{i_0 j_2}(\mathbf{x}(t))$ for some nonzero constant $c$. This at most implies that

$$[p_{i_0 j_1} - q_{i_0 j_1}(t)] + c[p_{i_0 j_2} - q_{i_0 j_2}(t)] = 0. \tag{13}$$

Clearly, although $q_{i_0 j_1}(t)$ and $q_{i_0 j_2}(t)$ are, respectively, identical to some constants $q_{i_0 j_1}^*$ and $q_{i_0 j_2}^*$, there exist infinite groups of constants $q_{i_0 j_1}^*$ and $q_{i_0 j_2}^*$ such that Eq. (13) is valid. Actually, the possibility of $q_{i_0 j_1}^* = p_{i_0 j_1}$ and $q_{i_0 j_2}^* = p_{i_0 j_2}$ is zero. The reason for failed parameter identification in case (b) could be easily illustrated likewise. Therefore, unknown parameter identification could not be physically realized if hypothesis [LIM] is not strictly satisfied [21].

Next, by virtue of the reasoning performed above, the reason why parameter identification fails in the preceding examples are explained as follows.

For the coupled systems (4) and (5) with the above specified parameters, the synchronized orbit $\mathbf{x}^*(t) = (x_1^*(t), x_2^*(t), x_3^*(t))^T$ in the synchronization manifold, as shown in Fig. 1, is an asymptotically stable equilibrium $E = \mathbf{x}^*(t) = (6.8313, 6.8313, 1.6667)^T$. Substitution of Eq. (4) into Eq. (12) gives

$$[p_1 - q_1(t)][x_2^*(t) - x_1^*(t)] = 0, \quad [p_2 - q_2(t)]x_1^*(t) = 0,$$

$$[p_3 - q_3(t)]x_3^*(t) = 0,$$

where each $q_i(t)$ is identical to some constant $q_i^*$ in the invariant set $\mathcal{M}$ $(i = 1,2,3)$. According to [20], each $x_i^*(t)(\neq 0)$ is linearly independent and $x_2^*(t) - x_1^*(t)$ $(\equiv 0)$ is linearly dependent. This implies that $q_{2,3}^*$ is identical to $p_{2,3}$, but $q_1^*$ is not necessarily identical to $p_1$. Therefore, $q_1(t)$, though obeying the updating law, will not be surely convergent to $p_1$. Now, the reason why parameter identification succeeds for $q_{2,3}$ and fails for $q_1$ in Fig. 2 is clear. In addition, when the synchronized orbit $\mathbf{x}^*(t)$ with the special parameters is chaotic, $x_2^*(t) - x_1^*(t)$ is not identical to zero. This nonzero property leads to a validity of hypothesis [LIM]. Hence, $q_1(t)$ is convergent to $p_1$ almost surely, which has been shown by many existing numerical results. Apart from the chaotic orbit, when the synchronized orbit $\mathbf{x}^*(t)$ unfortunately becomes an unstable equilibrium of the original chaotic systems, $x_2^*(t) - x_1^*(t)$ is still identical to zero. Also, this violation of hypothesis [LIM] leads to a failure of parameter identification for $q_1(t)$ with $p_1$.

For the coupled systems (6) and (7), the orbit $\mathbf{x}^*(t)$ in the synchronization manifold, as mentioned above, is designed to be a stable periodic orbit. Its projection into the $x_1$-$x_2$ plane, which seems similar to an ellipse, could be approximately expressed by the equation

$$\frac{[x_1\cos(0.9026) + x_2\sin(0.9026)]^2}{23.44^2} + \frac{[-x_1\sin(0.9026) + x_2\cos(0.9026)]^2}{7.19^2} - 1 = 0.$$

Analogously, substitution of Eq. (6) into Eq. (12) yields

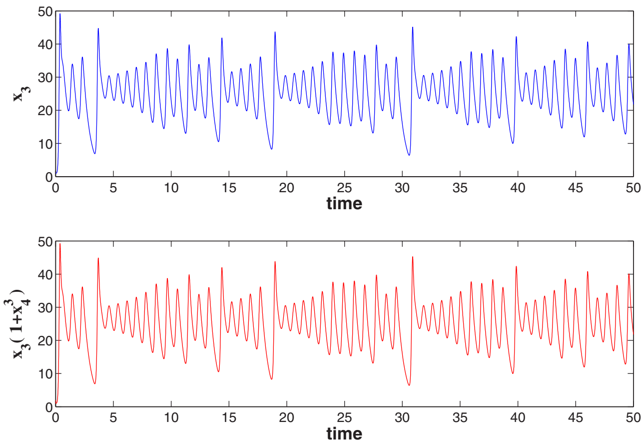


FIG. 7. (Color online) The variation of $x_3$ and $x_3(1+x_4^3)$ on the synchronized orbit $\mathbf{x}^*(t)$ with time, respectively.

$$[p_1 - q_1(t)]\frac{[x_1(t)\cos(0.9026) + x_2(t)\sin(0.9026)]^2}{23.44^2} + [-p_2 + q_2(t)] \times \left\{ \frac{[-x_1(t)\sin(0.9026) + x_2(t)\cos(0.9026)]^2}{7.19^2} - 1 \right\} = 0.$$

Thus, as long as the complete synchronization between systems (6) and (7) is achieved, the orbit $\mathbf{x}(t)$, as well as $\mathbf{y}(t)$, will approximately approach the stable periodic orbit $\mathbf{x}^*(t)$. Then, both functions

$$\frac{[x_1^*(t)\cos(0.9026) + x_2^*(t)\sin(0.9026)]^2}{23.44^2}$$

and

$$\frac{[-x_1^*(t)\sin(0.9026) + x_2^*(t)\cos(0.9026)]^2}{7.19^2} - 1$$

are approximately linearly dependent. This, according to the argument performed above, means that both $q_1$ and $q_2$ are not suitable for parameter identification, as is verified by the numerical results shown in Fig. 4.

Unlike the previous synchronized orbits, the orbit $\mathbf{x}^*(t)$ generated by system (8) is deliberately designed to be chaotic in the sense of possessing a positive Lyapunov exponent. Similarly, substitution of Eq. (8) into Eq. (12) produces

$$[p_3 - q_3(t)]x_3(t) + [p_4 - q_4]x_3(t)\{1 + [x_4(t)]^3\} = 0.$$

It is obvious that functions $x_3$ and $x_3(1+x_4^3)$ are linearly independent in the whole phase plane $\mathbb{R}^4$; nevertheless, they are approximately linearly dependent on the synchronized orbit $\mathbf{x}^*(t)$. This is because the cubic term $[x_4^*(t)]^3$ is almost equal to zero as time $t$ is sufficiently large (see Fig. 7). Thus, it illustrates the reason why $q_3$ and $q_4$, initiating from a mass of values, are not convergent to $p_3$ and $p_4$, respectively, in concrete numerical simulations.

In addition, consider a case that parameter $b$ in both systems (8) and (9) is selected to be zero instead of 0.1. In this case, because of $x_4^*(t) \equiv 0$, functions $x_3$ and $x_3(1+x_4^3)$ are definitely linearly dependent on the corresponding orbit $\mathbf{x}^*(t)$, which violates hypothesis [LIM]. Therefore, $q_3$ and $q_4$ cannot be utilized to identify the parameters $p_3$ and $p_4$. Although $x_4^*(t) \equiv 0$, the first three entries of $\mathbf{x}^*(t)$ still exhibit chaotic dynamics. Thus, this case could be regarded as a very special example where unknown parameter identification may fail in spite of the existence of chaos. In a word, chaotic property of the synchronized orbit in the synchronization manifold does not always guarantee an achievement of parameter identification.

*Remark*. In the last two examples, those functions on the orbits in the synchronization manifold are approximately linearly dependent. Mathematically, they are still linearly independent, so that unknown parameter identification could be theoretically achieved for each $q_i$ correspondingly with $p_i$. However, in real application, discretization techniques, such as the Runge-Kutta scheme and the Euler scheme, are always taken into account in solving the coupled continuous systems. Thus, owing to the precision limit, it is unavoidable that dynamics produced by the discretized systems may not be completely consistent with the true dynamics generated by the original systems. It is the approximate dependence of those functions that poses some trap of local critical point for $q_i$, and that leads to a failure of parameter identification in the last two examples. Therefore, *not only a rigorous linear dependence of functions with unknown parameters to be identified on the synchronized orbit but also an approximate linear dependence on the synchronized orbit should always be avoided whenever the adaptive synchronization techniques are used in practical parameter estimation and chaos communication.*

## IV. COMPLETE SYNCHRONIZATION WITHOUT GLOBAL LIPSCHITZ CONDITION

In the previous section, it is shown that hypothesis [LIM] is indispensable for a successful parameter identification. In addition, the uniform Lipschitz condition for $\mathbf{F}(\mathbf{x},\mathbf{p})$ is also crucial in the above-performed reasoning for obtaining a nonpositive property of $\dot{V}(\mathbf{e},\boldsymbol{\epsilon},\mathbf{q})$. As a matter of fact, this uniform condition could be loosed if the bounded property of the response system (3) could be priory estimated. However, this prior estimation could not directly follow from the bounded property of the driving system (1). This is because the dynamical evolution of the response system with additional coupling terms might be completely different from that of the driving system. Then, it poses another question: "Other than the above coupling technique and uniform Lipschitz condition, under what kind of coupling methods and conditions on $\mathbf{F}(\mathbf{x},\mathbf{p})$ can one obtain a successful parameter identification rigorously?"

To answer this question, we first assume that *each $F_i(\mathbf{x},\mathbf{p})$ is a homogeneous polynomial of $\mathbf{x}$ with a degree no more than two* [HPT]. Obviously, large quantities of nonlinear systems do not satisfy the global Lipschitz condition but are consistent with this assumption [HPT], such as the Lorenz system and the Chen system.

Next, notice that

$$2y_k y_j - 2x_k x_j = (y_k - x_k)(y_j + x_j) + (y_j - x_j)(y_k + x_k)$$
$$= 2e_k e_j + 2x_j e_k + 2x_k e_j$$

for arbitrary $k$ and $j$. Then, it is easy to verify that each $e_i[F_i(\mathbf{y},\mathbf{p}) - F_i(\mathbf{x},\mathbf{p})]$ can be written as a homogeneous polynomial of $\mathbf{e} = \mathbf{y} - \mathbf{x}$ with a degree no more than three if assumption [HPT] holds.

Reasonably, the driving signal $\mathbf{x}(t)$ generated by system (1) is supposed to be bounded in advance. In order to obtain a rigorous synchronization between the coupled systems where those nonlinear terms only satisfy assumption [HPT], we introduce a response system with some coupling terms in the form of

$$\dot{\mathbf{y}} = \mathbf{F}(\mathbf{y},\mathbf{q}) + \boldsymbol{\epsilon}\cdot\mathbf{e} + \boldsymbol{\omega}\cdot\mathbf{e}^3,$$

$$\epsilon_i = - r_i e_i^2, \quad \dot{\omega}_i = - s_i e_i^4,$$

$$\dot{q}_{ij} = - \delta_{ij} e_i f_{ij}(\mathbf{y}), \tag{14}$$

where $\boldsymbol{\omega}\cdot\mathbf{e}^3 = (\omega_1 e_1^3, \omega_2 e_2^3, \ldots, \omega_n e_n^3)^T$, each $s_i$ is arbitrarily positive constant, and other variables and parameters are the same as those defined in Eq. (3).

Set a Lyapunov function candidate by

$$H(\mathbf{e},\boldsymbol{\epsilon},\boldsymbol{\omega},\mathbf{q}) = \frac{1}{2}\sum_{i=1}^{n} e_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\delta_{ij}}(q_{ij} - p_{ij})^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{r_i}(\epsilon_i + M)^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{s_i}(\omega_i + N)^2.$$

Thus, the derivative of this function along with the coupled systems (1) and (14) yields

$$\dot{H}(\mathbf{e},\boldsymbol{\epsilon},\boldsymbol{\omega},\mathbf{q})(t) = \sum_{i=1}^{n} e_i(t)[F_i(\mathbf{y}(t),\mathbf{p}) - F_i(\mathbf{x}(t),\mathbf{p})] - \sum_{i=1}^{n} M e_i^2(t) - \sum_{i=1}^{n} N e_i^4(t),$$

where both $M$ and $N$ are positive numbers. From the conclusion on each $e_i[F_i(\mathbf{y},\mathbf{p}) - F_i(\mathbf{x},\mathbf{p})]$ obtained above, the elementary inequality

$$e_i e_j e_k \leqslant \frac{1}{6}\sum_{l=i,j,k}(e_l^2 + e_l^4),$$

and the assumed bounded properties of the driving signal $\mathbf{x}(t)$ and parameter set $\mathcal{U}$, it follows that $\dot{H}(\mathbf{e},\boldsymbol{\epsilon},\boldsymbol{\omega},\mathbf{q})(t) \leqslant 0$ for sufficiently large numbers $M$ and $N$.

By using the analogous reasoning performed in the preceding section, we can easily prove that every trajectory generated by the coupled systems (1) and (14) is not only bounded for all $t \geqslant t_0$ but also approaching the largest invariant set contained in

$$\mathcal{E}' = \{(\mathbf{x},\mathbf{y},\boldsymbol{\epsilon},\boldsymbol{\omega},\mathbf{q}) | \dot{H}(\mathbf{e},\boldsymbol{\epsilon},\boldsymbol{\omega},\mathbf{q}) = 0\}$$

with respect to these coupled systems. More precisely, for any trajectory $(\mathbf{e}(t), \boldsymbol{\epsilon}(t), \boldsymbol{\omega}(t), \mathbf{q}(t))$ contained in the largest invariant set $\mathcal{M}'$, we have

$$\mathbf{e}(t) \equiv 0, \quad \epsilon_i(t) \equiv \epsilon_i^*, \quad \omega_i(t) \equiv \omega_i^*, \quad q_{ij}(t) \equiv q_{ij}^*,$$

where $\epsilon_i^*$, $\omega_i^*$, and $q_{ij}^*$ are some constants dependent on the initial values of the coupled systems. Furthermore, to achieve an accurate parameter identification, hypothesis [LIM] should still be adopted. Then, the above performed argument could be concluded as the following proposition.

*Proposition 1.* If assumptions [LIM] and [HPT] on $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{p})$ are satisfied, the complete synchronization between the driving system (1) and its response system (14) could be surely achieved, and the parameter identification could be accurately realized in a mathematical sense.

*Remark 1.* As mentioned above, in numerical experiment and even in real application, not only hypothesis [LIM] should be strictly satisfied but also the approximate linear-dependence attributed to precision limit should be avoided.

*Remark 2.* Assumption [HPT] on $\mathbf{F}(\mathbf{x},\mathbf{p})$ could be further generalized to some other case where the global Lipschitz condition is not fulfilled. For instance, one could further consider the system where either the degree of the polynomials is above two, or where the polynomials are nonhomogeneous. However, additional coupling terms (e.g., $\boldsymbol{\kappa}_v \mathbf{e}^{2v+1}$, $v = 2, 3, \ldots$) should be added into the response systems in order to obtain a successful synchronization and parameter identification in a rigorous sense.

*Remark 3.* Note that those nonlinear terms in the previous three examples are not globally Lipschitz but polynomial. Thus, the theories and coupling techniques (i.e., proposition 1 and remark 2) proposed in this section ought to be utilized to deal with those systems for obtaining a successful synchronization and parameter identification.

## V. PARAMETER IDENTIFICATION IN SYSTEMS WITH TIME DELAYS

Time delay, as an omnipresent phenomenon, cannot be neglected in practice. So, in this section, complete synchronization and parameter identification in a system with time delays are further investigated. For simplicity, let us consider a one-dimensional driving system:

$$\dot{x}(t) = af(x(t)) + bg(x(t-\tau)), \tag{15}$$

where $\tau \geqslant 0$ is a time delay, $a$ and $b$ are parameters pending for identification, and functions $f$ and $g$ are assumed to be globally Lipschitz continuous with Lipschitz constants $k_f$ and $k_g$, respectively. Given the bounded driving signal $x(t)$ generated by system (15), the response system is designed to be in the form of

$$\dot{y}(t) = \alpha(t)f(y(t)) + \beta(t)g(y(t-\tau)) + \eta(t)e(t) + \omega(t)e(t-\delta),$$

$$\dot{\alpha}(t) = - f(y(t))e(t), \quad \dot{\beta}(t) = - g(y(t-\tau))e(t),$$

$$\dot{\eta}(t) = -e^2(t), \quad \dot{\omega}(t) = -e(t)e(t-\delta), \tag{16}$$

where $\delta \geqslant 0$ is a time delay induced by coupling term, error dynamics $e(t)=y(t)-x(t)$. The initial conditions for coupled systems (15) and (16) are chosen as $e=\phi=Y-X$, $\alpha=A$, $\beta=B$, $\eta=E$, and $\omega=W\in\mathcal{C}\triangleq\mathbb{C}([-\max\{\tau,\delta\},0],\mathbb{R})$, in which $\mathcal{C}$ denotes the sets of all continuous functions from $[-\max\{\tau,\delta\},0]$ to $\mathbb{R}$.

Set a Lyapunov functional candidate by

$$\begin{aligned}\mathcal{V}(\phi,A,B,E,W) &= \frac{1}{2}\phi^2(0) + \frac{1}{2}[A(0)-a]^2 + \frac{1}{2}[B(0)-b]^2 \\ &\quad + \frac{1}{2}[E(0)+L]^2 + \frac{1}{2}[W(0)+M]^2 \\ &\quad + \left\{\int_{-\tau}^{0} + \int_{-\delta}^{0}\right\}\phi^2(s)\mathrm{d}s,\end{aligned}$$

where $L$, $M$ are some proper positive constant. Then, the derivative of $\mathcal{V}$ along with coupled systems (15) and (16) could be estimated by

$$\begin{aligned}\dot{\mathcal{V}}(\phi,A,B,E,W) &\leqslant \left(ak_f + 2 - \frac{L}{2}\right)\phi^2(0) \\ &\quad + bk_g\cdot|\phi(0)|\cdot|\phi(-\tau)| - \phi^2(-\tau) \\ &\quad - \frac{L}{2}\phi^2(0) - M\phi(0)\phi(-\delta) - \phi^2(-\delta).\end{aligned}$$

Clearly, $\dot{\mathcal{V}}(\phi,A,B,E,W)$ becomes nonpositive provided $L>\max\{\frac{M^2}{2},\frac{b^2k_g^2}{2}+2ak_f+4\}$. By using a similar reasoning performed above, we can conclude that every trajectory $(x_t(X),y_t(Y),\alpha_t(A),\beta_t(B),\eta_t(E),\omega_t(W))$, starting from arbitrary initial condition, is surely bounded for all $t\geqslant-\max\{\tau,\delta\}$.

Then, according to the invariance principle for the systems with time delays [22], every trajectory, as time tends towards positive infinity, approaches the largest invariant set $\tilde{\mathcal{M}}$ contained in

$$\begin{aligned}\tilde{\mathcal{E}} = \{(X,Y,A,B,E,W) &\in \underbrace{\mathcal{C}\times\cdots\times\mathcal{C}}_{6}|\phi(0) \\ &= \phi(-\tau) = \phi(-\delta) = 0\}\end{aligned}$$

with respect to coupled systems (15) and (16). This further implies that the first two components of each element in $\tilde{\mathcal{M}}$ are identical (i.e., $\phi=Y-X\equiv0$) and the others are some constant functions (i.e., $A\equiv A^*$, $B\equiv B^*$, $E\equiv E^*$, and $W\equiv W^*$). The accurate values of these constant functions rest on the initial conditions of coupled systems (15) and (16).

Parameter identification could be achieved only if both equations $A^*=a$ and $B^*=b$ are valid. However, these equations are not always tenable though $\phi\equiv0$ indicates a successful complete synchronization between systems (15) and (16). In fact, subtraction of Eq. (15) from the first equation in Eq. (16) yields, in $\tilde{\mathcal{M}}$,

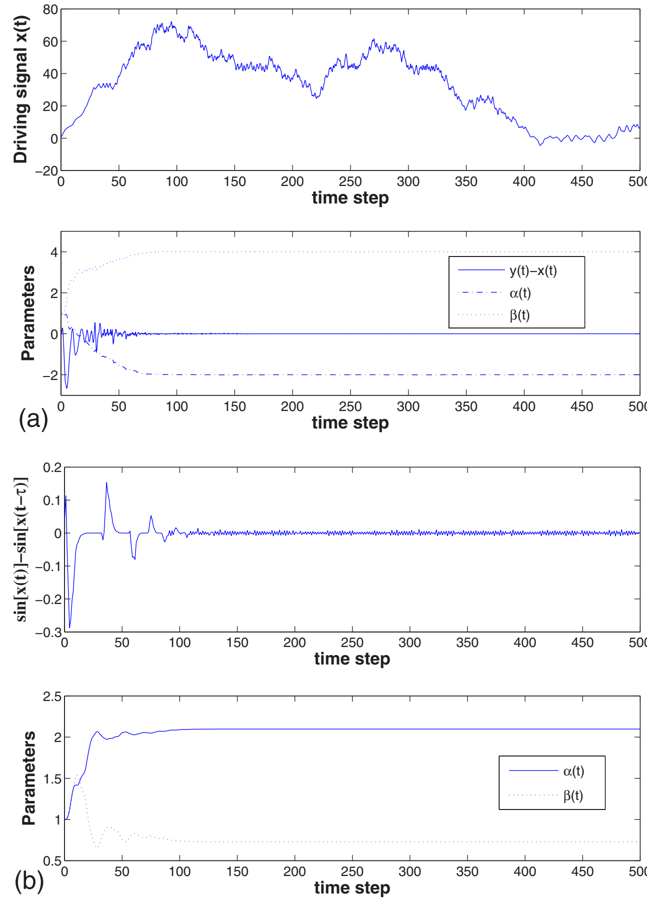


FIG. 8. (Color online) (a) Successful complete synchronization and parameter identification for chaotic driving signal when $a=-2$ and $b=4$; (b) failed parameter identification when $a=2$ and $b=1$. This failure is simply due to an approximate dependence between $f(x(t))$ and $g(x(t-\tau))$ in a macroscale. Here, both $f$ and $g$ are taken as sinusoid functions, time delays are taken as $\tau=10$, $\delta=2$, and time step size is 0.01.

$$\begin{aligned}0 &= \alpha(t)f(y(t)) - af(x(t)) + \beta(t)g(y(t-\tau)) - bg(x(t-\tau)) \\ &= [\alpha(t)-a]f(x(t)) + [\beta(t)-b]g(x(t-\tau)) \\ &= [A^*-a]f(x(t)) + [B^*-b]g(x(t-\tau)),\end{aligned}$$

which follows from $e(t)=y(t)-x(t)\equiv0$ in $\tilde{\mathcal{M}}$. Now, it is clear that if functions $f(x(t))$ and $g(x(t-\tau))$ are linearly dependent on the synchronized orbit $x(t)$ in the synchronization manifold, the probability of $A^*=a$ and $B^*=b$ is certainly equal to zero. More precisely, (i) if the driving signal asymptotically tends towards some equilibrium of system (15), namely $x(t)\equiv x^*$, two constant functions $f(x(t))\equiv f(x^*)$ and $g(x(t-\tau))\equiv g(x^*)$ becomes linearly dependent, so that the parameter identification for $a$ and $b$ is almost surely failed; (ii) if the synchronized orbit $x(t)$ is periodic with period $\tau$ and both functions $f$ and $g$ are linearly dependent in $\mathbb{R}$, parameter identification will be failed; (iii) if $x(t)$ is chaotic, parameter identification will be achieved theoretically for nonconstant differential functions $f$ and $g$, and even for $f=g$ [see an example shown in Fig. 8(a) where both $f$ and $g$ are taken as sinusoid functions]. However, in case (iii), parameter identification might also be failed in numerical simulation or in real application. Examples abound: the fluctuation of $x(t)$, though chaotic, possibly seems to be relatively steady in a macroscale; $x(t) \approx x(t-\tau)$ where time delay is comparatively small. These extraordinary cases may lead to an approximate linear-dependence between functions $f(x(t))$ and $g(x(t-\tau))$, which thus results in a failure of parameter identification in numerical simulation. See an illustrative example shown in Fig. 8(b). In addition, $\tau=0$ could be regarded as a special case where parameter identification is always failed, provided that functions $f$ and $g$ are linearly dependent on $x(t)$.

In conclusion, we have the following proposition on synchronization and parameter identification for coupled systems (15) and (16) with time delay.

*Proposition*. The complete synchronization between the driving system (15) and its response system (16) could be achieved via adaptive coupling techniques. Furthermore, the parameter identification could be accurately realized in a mathematical sense, provided that $f(x(t))$ and $g(x(t-\tau))$ are linearly independent on the synchronized orbit $x(t)$ in the synchronization manifold.

*Remark*. With an analogous reasoning but more complicated notations, the results on the driving system (15) could be further generalized to the case where higher dimensional driving systems and multiple parameter identifications are taken into account. However, the linear independence of all the functions with unknown parameters on the synchronized orbit is crucial to a successful parameter identification.

## VI. CONCLUSION

In summary, concrete examples showing possible occurrence of failed parameter identification have been numerically provided in the paper. The reason for this failure has been studied. It has been shown that the chaotic property of the driving system is not always crucial to an achievement of unknown parameter identification either in mathematical reasoning or in numerical experiment. Actually, it is not the chaos but the hypothesis [LIM] that guarantees a success in parameter identification based on adaptive synchronization techniques. However, making good use of the chaotic property might easily lead to a validity of hypothesis [LIM]. Apart from the linear dependence of functions with unknown parameters on the synchronized orbit, the approximate linear dependence should also be avoided in numerical simulation and even in real application. In addition, it has been rigorously verified that every trajectory generated by the coupled system is bounded.

Furthermore, in light of the LaSalle invariance principle and the particular properties of autonomous system, complete synchronization in a class of polynomial systems where nonlinear terms are not globally Lipschitz has been theoretically investigated. By all these derived theoretical results, our proposed coupling technique is proven to be a rigorous and feasible approach for realization of complete synchronization and unknown parameter identification in the Lorenz-like systems. Besides, adaptive coupling technique is imported to realize unknown parameter identification in systems with time delay. Those discussions also show the great importance of the condition that functions with unknown parameters should be linearly independent on the synchronized orbit. Since the theoretical results obtained in the paper are based on the properties of autonomous system, unknown parameter identification in nonautonomous system could be further analytically studied, leaving some interesting topics for our future investigation.

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[21] If there exist linearly dependent functions on the synchronized orbit, the varible $\mathbf{q}(t)$ is not surely convergent. However, in some particular cases, the convergence of $\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\delta_{ij}}(q_{ij}-p_{ij})^2$ might be useful in determination of the convergence of $\mathbf{q}(t)$.
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